Letting Time Lie Down: Visualizing and Quantifying Simple Harmonic Motion

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Simple harmonic motion (SHM) is easy to watch but hard to see as a sinusoid: the back-and-forth happens on a single line, and the displacement–time graph lives only in textbooks. We present a three-step teaching sequence that unfolds the time axis into a spatial dimension, making the abstraction concrete. The sequence moves from a physical analog demonstration to a rapid digital analysis, culminating in a rigorous class-wide quantitative test. **Step 1 — see it.** A violet laser fastened to a spring–mass oscillator writes a glowing sinusoid on a phosphorescent foil carried past it by a slowly rolling cart. **Step 2 — slice it.** A free browser tool, the *Frame Sequencer*, tiles vertical strips of any smartphone video into the same kind of $y(t)$ image and reports a sinusoidal fit to four significant figures in about thirty seconds. **Step 3 — solve it.** Student-recorded videos spanning a range of masses and two spring configurations are pooled into a single T^2-versus-m plot whose measured slope ratio of 1.99 ± 0.06 confirms the SHM prediction $T^2 \propto m/k_{\text{eff}}$ to within 1 %. The full sequence runs on equipment a school already owns or can buy for the price of a textbook.

The problem of "seeing" SHM

Simple harmonic motion sits at the heart of the introductory physics curriculum, and students meet it again whenever they study waves, sound, or AC circuits. Unlike projectile motion, whose parabolic trajectory already gives students a two-dimensional picture, one-dimensional oscillation offers only a back-and-forth along a single line, and the canonical sinusoidal x–t graph is not actually visible in the laboratory. McDermott, Rosenquist, and van Zee documented long ago that translating between motion and graphs is a persistent student difficulty,[1] subsequently confirmed by Beichner.[2] Existing remedies include low-cost mechanical tracers,[3] Arduino-based distance sensors,[4] smartphone accelerometers,[5] video-analysis packages such as Tracker,[6] and phosphorescent-foil displays for Lissajous figures.[7]

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Step 1 — See it: a glowing sinusoid on the bench

The first step is a sensory demonstration: a simple analog apparatus that lets students watch a sinusoid being drawn in real time (Fig. 1).

Apparatus. Two identical springs of stiffness k are suspended in parallel from a retort stand; the parallel arrangement keeps the mass from rotating about a vertical axis as it oscillates. A small violet laser pointer (405 nm, < 1 mW, Class 2, 21 CFR compliant) is rigidly attached beneath the mass via a 3D-printed bracket with the beam horizontal; as with any Class 2 source, students are instructed not to look directly into the beam.

Physical time axis. An A4 phosphorescent foil is mounted vertically on a low-friction toy car towed along the bench by a small electric motor at a constant horizontal speed of a few centimeters per second. With two identical springs in parallel the effective stiffness is $2k$; the suspended hook mass is then adjusted so that the oscillation period is about 1 s, convenient for observation at typical cart speeds.

Observations. As the cart moves horizontally (the time axis) and the oscillator moves vertically (the displacement axis), the laser briefly excites the phosphor and leaves a glowing afterimage that traces out a clean sinusoid (Fig. 1). The trace persists for nearly a minute. Students see the sinusoidal shape emerge in front of them before any equation is written or any number measured. A short video of the recorder in operation is provided as Supplementary Material.

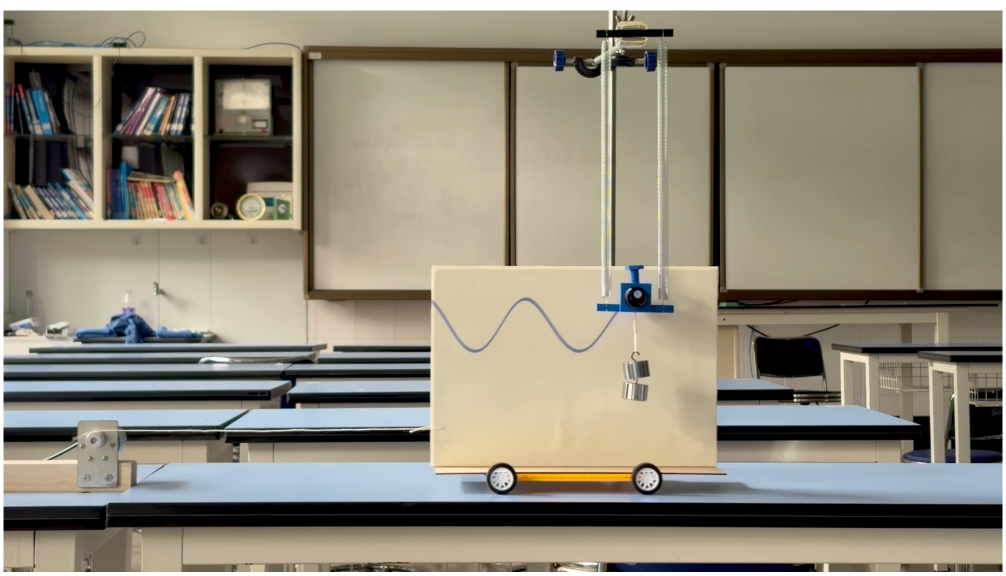

Figure 1: The laser–phosphor recorder during a run. A violet laser pointer below a parallel-spring oscillator writes a glowing sinusoid on the phosphorescent foil mounted on a cart towed along the bench at a few centimeters per second.

Step 2 — Slice it: every phone is an oscilloscope

Once the connection between SHM and the sinusoid has been seen, students need a tool to capture the same motion quantitatively. We built the *Frame Sequencer* (Fig. 2), a free, browser-based tool that does in software what the cart does in hardware. The live version is hosted at `https://frame-sequencer-909934852596.us-west1.run.app` and is actively maintained by the authors; a stable offline build that will not be updated further, a screen recording of the workflow, and the sample spring–mass video used in this section are provided as Supplementary Material.

Principle. As in Step 1, time itself is mapped to a spatial axis: vertical strips of a video are tiled side by side along the screen so that the motion writes its own x–t graph. The Frame Sequencer runs entirely in a browser with no installation, automatically detects the region of motion and the moving object, and automatically fits a chosen mathematical model (linear, quadratic, or sinusoidal) to the extracted trajectory.

Workflow. In practice, the user uploads a short smartphone clip; 5 to 10 seconds is usually enough. To minimize camera shake from pressing the record button, the tool defaults to a 3-second window centered within the clip, sampled at 15 fps; the window length, position, and sampling rate are user-adjustable. A multi-frame overlay (top of Fig. 2) is used to locate the region of significant motion automatically (purple box), with manual selection also supported. Each sampled frame is cropped to that region, and the cropped strips are tiled side by side in chronological order. Because every strip has the same width and the frames are sampled at equal time intervals, the horizontal coordinate maps linearly to physical time t via the sampling rate, and the vertical axis corresponds to position, so the composite is a true displacement–time image $y(t)$ rather than a displacement–pixel plot. The tool detects the moving object in each strip (red dots in Fig. 2) and overlays a best-fit curve (green) of the chosen model, returning the equation and R^2.

Observations. The bottom panel of Fig. 2 is the composite produced from a sample spring–mass video. The periodic pattern is obvious by eye, and the sinusoidal fit returns

$$y(t) = 229.31\ \sin(8.1263\,t + 3.72) + 1200.33, \tag{1}$$

with $R^2 = 0.9967$, corresponding to a period $T = 2\pi/\omega \approx 0.77$ s. A few seconds of phone footage suffice to measure the period accurately, with no installation and no need to step through frames by hand. The full pipeline takes about thirty seconds, fast enough for dozens of runs within a single class.

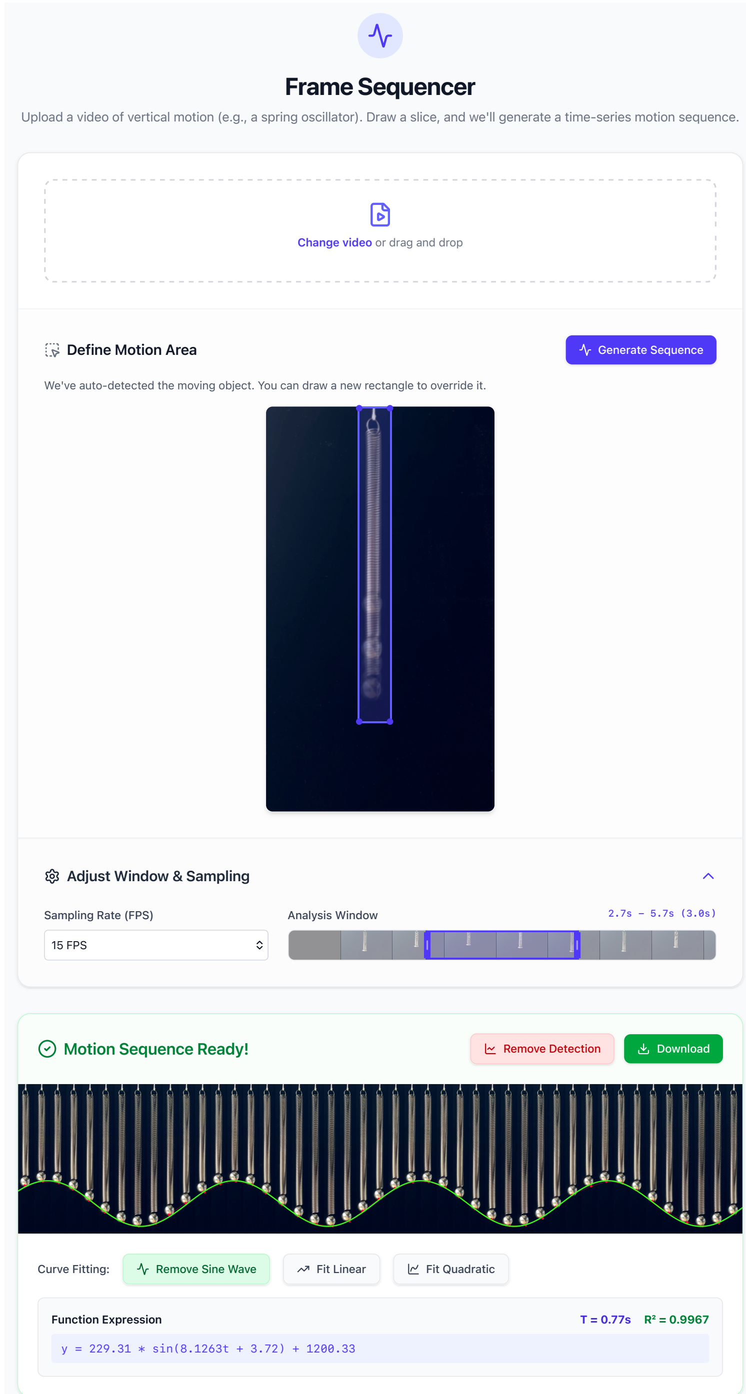

Figure 2: The Frame Sequencer workflow. Top: a multi-frame overlay of the uploaded video, with the moving object detected automatically (purple box). Middle: a 3-s window sampled at 15 fps. Bottom: cropped strips tiled in time order, the object tracked across strips (red dots), and a sinusoidal fit (green) returning $y(t) = A\sin(\omega t + \varphi) + c$ with T and R^2.

Step 3 — Solve it: the class as collaborators

With a tool that returns a numerical period from any video, it becomes practical for a whole class to test the SHM prediction quantitatively, working as a team. Students in one author's class recorded smartphone videos under the instructor's guidance, in two spring configurations: a single spring (supplier nominal $k = 24$ N/m), and two identical springs in series ($k_{\text{eff}} = k/2$). The suspended mass was varied from 150 g to 400 g in 50-g steps. For each mass-and-spring combination the video was uploaded to the Frame Sequencer and the period T read off from the automatic sinusoidal fit; most fits returned $R^2 > 0.99$, most exceeding 0.997. The composite of all outputs is shown in Fig. 3: within each block the period T grows as m increases, and halving the effective stiffness (upper block to lower block) lengthens T further by a factor of $\sqrt{2}$.

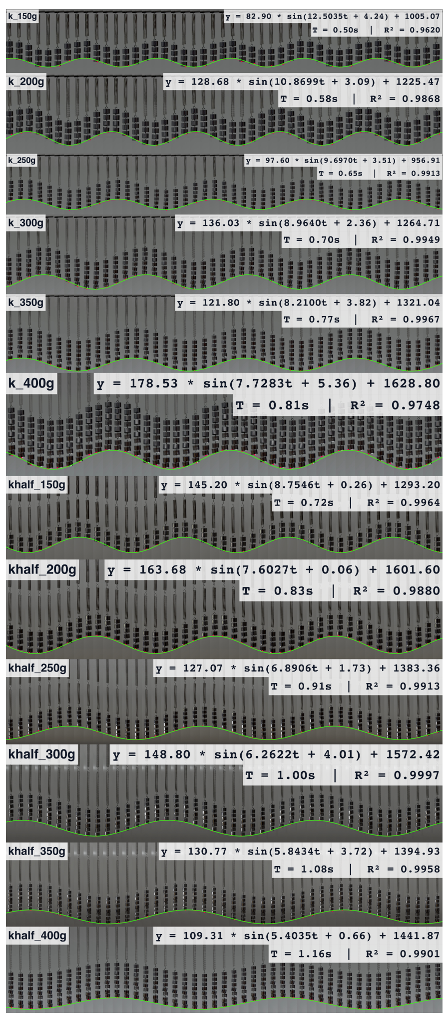

Figure 3: Combined Frame-Sequencer outputs for the student videos, organized by spring configuration and by suspended mass. Each panel shows the time-stacked composite with the detected position (red) and the best-fit sinusoid (green); the spring configuration and mass are labeled in the upper-left corner, and the fitted equation together with the extracted period T and R^2 are shown in the upper-right corner.

Linear model. For an oscillator of mass m on springs of effective stiffness k_{eff}, simple harmonic motion predicts

$$T^2 = \frac{4\pi^2}{k_{\text{eff}}} m, \tag{2}$$

so T^2 versus m should fall on a straight line with slope $4\pi^2/k_{\text{eff}}$. The pooled student data are plotted in Fig. 4.

Linearity. Both configurations sit on clean straight lines, with $R^2 = 0.9974$ (single spring) and $R^2 = 0.9989$ (series pair). The high R^2 values confirm the predicted $T^2 \propto m$ relationship across the full mass range.

Slope ratio. Going from a single spring ($k_{\text{eff}} = k$) to two identical springs in series ($k_{\text{eff}} = k/2$) should multiply the slope by exactly 2.00. The measured slope ratio is 1.99 ± 0.06, matching the theoretical value to within 1 %.

Non-zero intercept and the spring's own weight. Both fitted lines show small positive intercepts rather than passing exactly through the origin. This is consistent with the mass of the spring itself: a spring of mass m_{s} contributes an effective mass $m_{\text{s}}/3$ to the oscillator,[8] so Eq. (2) generalizes to

$$T^2 = \frac{4\pi^2}{k_{\text{eff}}} \left(m + \frac{m_{\text{s, eff}}}{3} \right), \tag{3}$$

shifting the line upward by $(4\pi^2/k_{\text{eff}})(m_{\text{s, eff}}/3)$ at $m = 0$. The series configuration, with halved k_{eff} and twice the active spring material, has the larger observed intercept, in qualitative agreement with Eq. (3).

Consistency check — recovering k. Inverting the fitted slopes through $k_{\text{eff}} = 4\pi^2/\text{slope}$ provides an independent measurement of the spring constant from each configuration. The single-spring slope of $1.65\,\text{s}^2/\text{kg}$ gives $k \approx 23.9$ N/m; the series-pair slope of $3.28\,\text{s}^2/\text{kg}$ gives $k_{\text{eff}} \approx 12.0$ N/m, hence $k \approx 24.1$ N/m. The two values agree with each other and with the supplier's nominal $k = 24$ N/m to within $\sim 0.5\,\%$, providing an internally consistent verification of the SHM model.

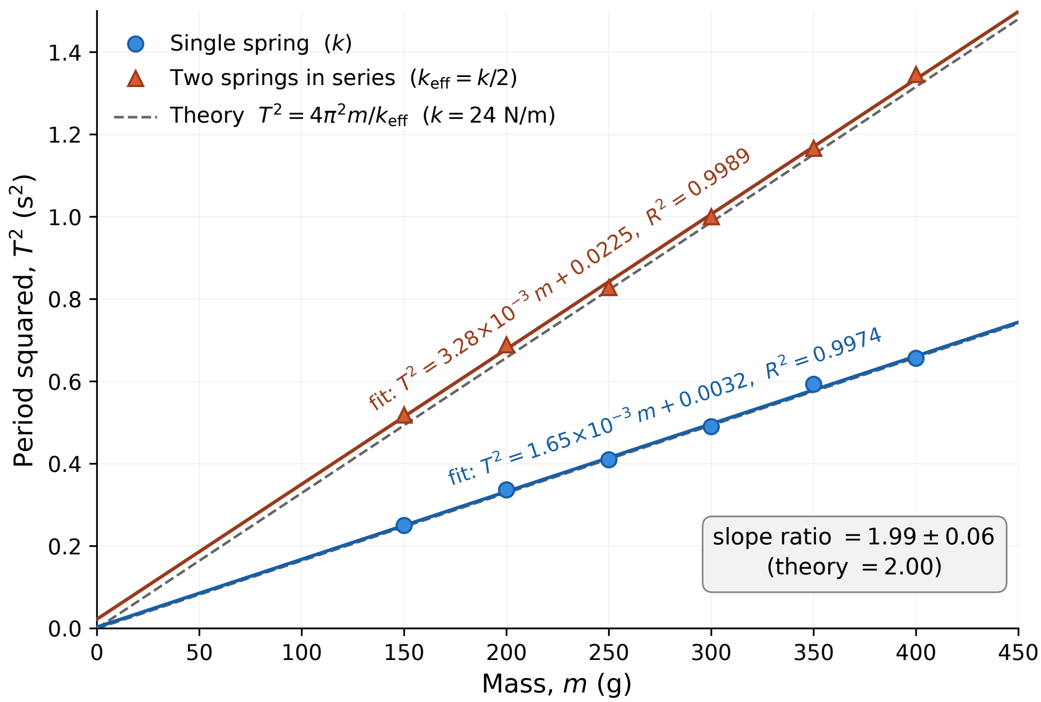

Figure 4: Period squared T^2 versus suspended mass m for the single-spring (blue circles) and two-springs-in-series (orange triangles) configurations. Solid lines are linear fits to the data ($R^2 = 0.9974$ and 0.9989); dashed gray lines are the parameter-free SHM predictions $T^2 = 4\pi^2 m/k_{\text{eff}}$ obtained by substituting the supplier's nominal $k = 24$ N/m into Eq. (2), not fitted to the data.

Supplementary Material

Readers can access the supplementary material at TPT Online. It contains:

- `analog_demo.mov` — the laser–phosphor recorder of Step 1 in operation.
- `frame-sequencer.html` — a self-contained, offline-runnable build of the Frame Sequencer (Step 2), frozen as a stable snapshot that will not be updated further; open in any modern browser, no installation.
- `frame_sequencer_workflow.mov` — a short screen recording of the Frame Sequencer workflow.
- `spring_mass_sample.mov` — the sample spring–mass video used to produce Fig. 2.

References

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